# Relay Feedback Method for Processes Under Noisy Environments

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# Introduction

Since Astrom and Hagglund<sup>1</sup> introduced the relay feed-back method, many variations have been proposed for autotuning of PID controllers.<sup>2–4</sup> Recently, it was shown that integrals of relay feedback responses can reduce errors in estimating process dynamics<sup>5</sup> by suppressing the higher harmonic terms.

For noisy environments, the ideal relay suffers from chattering. This chattering does not affect characteristics of relay oscillation such as stability. However, for some processes, chattering is not allowed. To avoid this chattering, a relay with hysteresis can be used. However, its switching period differs from the ultimate period.<sup>2</sup> Alternatively, a low-pass filter in the feedback loop may be used, but it also suffers from the same problem due to the phase lag of filter. In this research, a chattering-free relay feedback method for processes under noisy environment is proposed. It provides the ultimate period without offset.

# **Relay with Hysteresis**

Consider a linear process

$$Y(s) = G(s)U(s) \tag{1}$$

When the process output is corrupted by noise, the standard relay shows chattering. To avoid this, a relay with hysteresis can be used (Figure 1a), where the relay is switched on when e(t) is greater than  $e_h$  and is switched off when e(t) is less than  $-e_h$  ( $e_h$  is the magnitude of relay hysteresis). For the process

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 $G(s) = 5/(s + 1)^6$ , typical responses of the above relay feedback system are shown in Figure 1b. From them, we can obtain an average period of oscillation, p, and amplitude,  $a_y$ . Then, the oscillation period is such that

$$G(j2\pi/p) \approx -\frac{\pi}{4d} \left( \sqrt{a_y^2 - e_h^2} + je_h \right)$$
 (2)

It differs from the ultimate data as shown in Figure 1c. Furthermore, the noise makes it difficult to find the period and amplitude of oscillation accurately.

# **Proposed Method**

A chattering-free method that provides the ultimate period without offset is proposed. We use the process output y(t) and its integral q(t). The relay is switched as shown in Figure 2.

Step 1: The relay output is kept -h when the process output is positive.

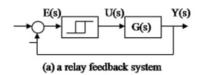
Step 2: The relay output is switched to zero when the process output corrupted with noise crosses zero first (the white circle in Figure 2). The value of q(t) is saved at that time. The relay output is kept at zero until q(t) crosses the saved value (the filled circle in Figure 2).

Step 3: The relay output is switched to h when q(t) crosses the saved value (the filled circle in Figure 2) and is maintained for the negative process output.

Step 4: The relay output is switched to zero when the process output crosses zero first (the white square in Figure 2). Save the value of q(t) at that time. The relay output is kept at zero until q(t) crosses the saved value (the filled square in Figure 2).

Step 5: Repeat Steps 1–4 for a given number of oscillations.

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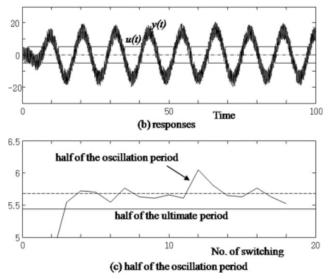


Figure 1. Responses of a feedback system of relay with hysteresis  $(G(s) = 5/(s + 1)^6)$ .

Let  $t_1$  and  $t_2$  be durations of relay outputs of zero and on(off), respectively. The Fourier series of the relay output u(t) is  $^6$ 

$$u(t) = \frac{4h}{\pi} \left( \cos\left(\frac{t_1\pi}{p}\right) \sin\left(\frac{2\pi}{p}t\right) + \frac{1}{3}\cos\left(3\frac{t_1\pi}{p}\right) \sin\left(3\frac{2\pi}{p}t\right) + \cdots \right)$$
$$p = 2(t_1 + t_2) \tag{3}$$

where h is the relay output size and p is the oscillation period. When the average amplitude of the process output,  $a_y$ , is measured, we have the ultimate gain,

$$K_{\rm cu} = \frac{4h \, \cos\left(\frac{t_1 \pi}{p}\right)}{\pi a_y} \tag{4}$$

However,  $a_y$  is difficult to measure because of noise. Instead of  $a_y$ , we can use the amplitude of the integral of the process output  $a_q$ , 5

$$K_{\rm cu} = \frac{2hp \, \cos\left(\frac{t_1\pi}{p}\right)}{\pi^2 a_q} \tag{5}$$

The ratio of the third harmonic term over the first harmonic term is

$$\gamma = \frac{1}{3} \cos \left( 3 \frac{t_1 \pi}{p} \right) / \cos \left( \frac{t_1 \pi}{p} \right) \tag{6}$$

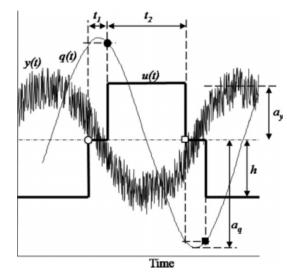
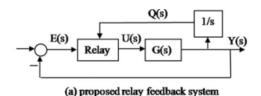


Figure 2. Responses of the proposed relay feedback system (y(t): process output, q(t): integral of the process output, and u(t): process input).

Its amplitude is smaller than that of the original relay  $(t_1 = 0)$  for  $0 < t_1 < p/4$ . Hence, the proposed method can also reduce errors in the ultimate period identification.

The proposed method is simulated with the process  $G(s) = 5/(s+1)^6$ . Responses are shown in Figure 3. We can see that an accurate ultimate period is obtained.



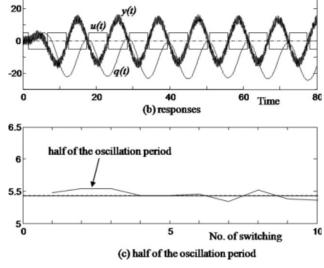


Figure 3. Responses of a proposed relay feedback system  $(G(s) = 5/(s + 1)^6)$ .

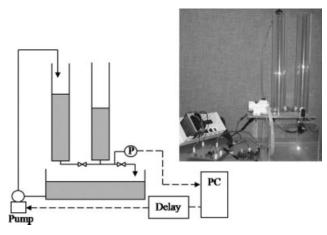


Figure 4. An experimental liquid level process.

# **Experimented Validation**

The proposed method is applied to an experimental liquid level process shown in Figure 4. Two tanks are connected in an interactive manner. A small pump with a pulse width modulation input is used to control the feed flow rate. Process input and output variables are interfaced with the National Instruments data acquisition system using LabView control software. Because the two tank system is a second-order system and is not adequate for the relay feedback experiment, delay of 4.5 s is added artificially to the signal to control the pump. Figure 5 shows experimental responses. The exact ultimate period is not known because it is changing due to inaccurate measurements. However, we can see that the fluctuation in the obtained oscillation period is very small.

#### Conclusion

A simple method to obtain the ultimate period and the ultimate gain of process under noisy environments is proposed. Both simulation and experimental results indicate that the method provides improved results over existing methods.

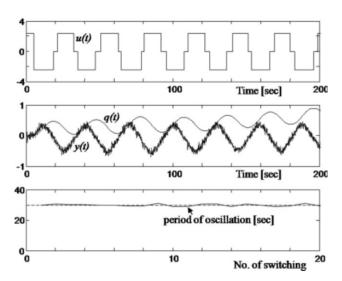


Figure 5. Responses of the proposed relay feedback system for the experimental liquid level process.

# **Acknowledgments**

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